## 3.5 Composition Functions MATH 1610 THOMPSON

1) The domain of the composite function $(f \circ g)(x)$ is the NOT the same as g(x). False

2) Find $(f \circ g)(x)$ if $f(x) = \sqrt{x+4}$ and $g(x) = \frac{2}{x}$ we plug the g into f $\sqrt{\frac{2}{x}+4}$

3) If $H = f \circ g$ and $H(x) = \sqrt{36-16x^2}$, which of the following cannot be the component functions f and g?

$f(x) = \sqrt{36-x^2}$; $g(x) = 16x$

4) Given $f(x) = 5x$ and $g(x) = 4x^2 + 4$, find the following expressions.

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

**Plug the # into the inner function then plug that answer into the outer function**

$g(4) = 4(4)^2 + 4 = 68$ $f(2) = 5(2) = 10$ $f(1) = 5(1) = 5$ $g(0) = 4(0)^2 + 4 = 4$

$f(68) = 5(68) =$ **340** $g(10) = 4(10)^2 + 4 =$ **404** $f(5) = 5(5) =$ **25** $g(4) = 4(4)^2 + 4 =$ **68**

5) Given $f(x) = 2x^2 - 1$ and $g(x) = 6 - \frac{1}{2}x^2$, find the following expressions.

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

$g(4) = 6 - \frac{1}{2}(4)^2 = -2$ $f(2) = 2(2)^2 - 1 = 7$ $f(1) = 2((1)^2 - 1 = 1$ $g(0) = 6 - \frac{1}{2}(0)^2 = 6$

$f(-2) = 2(-2)^2 - 1 =$ **7** $g(7) = 6 - \frac{1}{2}(7)^2 = -\frac{37}{2}$ $f(1) = 2(1)^2 - 1 =$ **1** $g(6) = 6 - \frac{1}{2}(6)^2 =$ **-12**

6) For $f(x) = 5x + 4$ and $g(x) = 8x$, find the following composite functions and state the domain of each.

**Write the function listed first with a blank in place of the x value**

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

**5( ) + 4** **8( )** **5( ) + 4** **8( )**

5(8x) + 4 8(5x+4) 5(5x+4) + 4 8(8x)

40x + 4 40x+32 25x + 24 64x

B. The domain of $g \circ f$ is all real numbers. For all parts because they are lines

7) For $f(x) = 7x + 9$ and $g(x) = x^2$, find the following composite functions and state the domain of each.

(a) $f \circ g$ $7(x^2) + 9$ $7x^2 + 4$

(b) $g \circ f$ $(7x+9)^2$ $49x^2+126x+81$

(c) $f \circ f$ $7(7x+9) + 9$ $49x + 72$

(d) $g \circ g$ $(x^2)^2$ $x^4$

B. The domain of $g \circ f$ is all real numbers. For all parts because they are parabolas

8) For $f(x) = x^2$ and $g(x) = x^2 + 7$, find the following composite functions and state the domain of each.

(a) $f \circ g$ $(x^2+7)^2$ $x^4+14x^2+49$

(b) $g \circ f$ $(x^2)^2+7$ $x^4+7$

(c) $f \circ f$ $(x^2)^2$ $x^4$

(d) $g \circ g$ $(x^2+7)^2+7$ $x^4+14x^2+56$

B. The domain of $g \circ f$ is all real numbers. For all parts because they are parabolas

9) $f(x) = \frac{8}{x-3}$ $g(x) = \frac{1}{x}$

***multiply the single term (3) in the denominator by x because of $\frac{1}{x}$

(a) $f \circ g = \frac{8}{\left(\frac{1}{x}\right)-3} = \frac{8}{\left(\frac{1}{x}\right)-3(x)} = \frac{8}{\frac{1-3x}{x}}$ =flip bottom and multiply= $\frac{8x}{1-3x}$

Domain $\{x|x \neq 0, \frac{1}{3}\}$ check **both** denominators (answer and original)

(b) $g \circ f$ $\frac{1}{\left(\frac{8}{x-3}\right)} =$ *flip bottom and multiply* $= \frac{x-3}{8}$

Domain $\{x|x \neq 3\}$ only original has restrictions

(c) $f \circ f = \frac{8}{\left(\frac{8}{x-3}\right)-3} = \frac{8}{\left(\frac{8}{x-3}\right)-3(x-3)} = \frac{8}{\frac{8-3x+9}{x-3}} = \frac{8}{\frac{17-3x}{x-3}}$ =flip bottom and multiply= $\frac{8(x-3)}{17-3x}$

Domain $\{x|x \neq 3, \frac{17}{3}\}$

(d) $g \circ g = \frac{1}{\left(\frac{1}{x}\right)} =$ flip bottom and multiply = x

Domain $\{x|x \neq 0\}$ only original has restrictions

10) $f(x) = \frac{x}{x-4}$ $g(x) = \frac{-7}{x}$

a) Find f°g(x) $= \frac{(\frac{-7}{x})}{(\frac{-7}{x})-4} = \frac{(\frac{-7}{x})}{(\frac{-7}{x})-4(x)} = \frac{\frac{-7}{x}}{\frac{-7-4x}{x}} = \frac{-7}{-7-4x}$

Domain $\{x|x \neq 0, -\frac{7}{4}\}$ check both denominators (answer and original)

b) Find g°f(x) $= \frac{-7}{(\frac{x}{x-4})} =$ *flip bottom and multiply* $\frac{-7(x-4)}{x} = \frac{-7x+28}{x}$

Domain $\{x|x \neq 4,0\}$ check both denominators

c) Find f°f(x) $= \frac{\frac{x}{x-4}}{(\frac{x}{x-4})-4} = \frac{\frac{x}{x-4}}{(\frac{x}{x-4})-4(x-4)} \frac{\frac{x}{x-4}}{\frac{x-4x+16}{x-4}} = = \frac{x}{-3x+16}$

Domain $\{x|x \neq 4, \frac{16}{3}\}$

d) Find g°g(x) $= \frac{-7}{(\frac{-7}{x})} =$ *flip bottom and multiply* $\frac{-7(x)}{-7}$ = x

Domain $\{x|x \neq 0$ only original has restrictions

****** DOMAIN for $\sqrt{x}$** is $x \geq 0$

11) f(x) = $\sqrt{x}$ g(x) = 6x+1

a. Find f°g(x) $= \sqrt{6x+1}$ Domain $\{x|x \geq -\frac{1}{6}\}$

b. Find g°f(x) $= 6\sqrt{x}+1$ Domain $\{x|x \geq 0\}$

c. Find f°f(x) $= \sqrt[4]{x}$ Domain $\{x|x \geq 0\}$

d. Find g°g(x) $= 6(6x+1)+1 = 36x+7$ Domain all reals

12) $f(x) = x^2 + 6$ $g(x) = \sqrt{x-4}$

a. Find $f\circ g(x) = (\sqrt{x-4})^2 + 6 = x + 2$ Domain $\{x|x \geq 4\}$

b. Find $g\circ f(x) = \sqrt{x^2+6-4} = \sqrt{x^2+2}$ Domain all reals

c. Find $f\circ f(x) = (x^2+6)^2 + 6 = x^4 + 12x^2 + 42$ Domain all reals

d. Find $g\circ g(x) = \sqrt{\sqrt{x-4}-4}$ Domain $\{x|x \geq 20\}$

**Square 4 = 16 then add 4 (16+4) = 20

*ALWAYS SQUARE THE # THEN ADD IT TO ITSELF*

*Another example:* $\sqrt{\sqrt{x-8}-8} \rightarrow 8^2 + 8 = 75$ *THEN Domain* $\{x|x \geq 72\}$

13) $f(x) = \frac{x-4}{x+7}$ $g(x) = \frac{x+2}{x-3}$

a) Find $f\circ g(x) = \frac{\left(\frac{x+2}{x-3}\right)-4}{\left(\frac{x+2}{x-3}\right)+7} = \frac{\frac{x+2-4(x-3)}{x-3}}{\frac{x+2+7(x-3)}{x-3}} = \frac{x+2-4x+12}{x+2+7x-21} = \frac{-3x+14}{8x-19}$

**bring the x+2 over then distribute the 7 with the denominator

Domain $\{x|x \neq 3, \frac{19}{8}\}$ check both denominators **(original and answer)**

b) Find $g\circ f(x) = \frac{\left(\frac{x-4}{x+7}\right)+2}{\left(\frac{x-4}{x+7}\right)-3} = \frac{\frac{x-4+2(x+7)}{x+7}}{\frac{x-4-3(x+7)}{x+7}} = \frac{x-4+2x+14}{x+-4-3x-21} = \frac{3x+10}{-2x-25}$

Domain $\{x|x \neq -7, -\frac{25}{2}\}$ check both denominators

c) Find $f\circ f(x) = \frac{\left(\frac{x-4}{x+7}\right)-4}{\left(\frac{x-4}{x+7}\right)+7} = \frac{\frac{x-4-4(x+7)}{x+7}}{\frac{x-4+7(x+7)}{x+7}} = \frac{x-4-4x-28}{x-4+7x+49} = \frac{-3x-32}{8x+45}$

Domain $\{x|x \neq -7, -\frac{45}{8}\}$ check both denominators

d) Find $g\circ g(x) = \frac{\left(\frac{x+2}{x-3}\right)+2}{\left(\frac{x+2}{x-3}\right)-3} = \frac{\frac{x+2+2(x-3)}{x-3}}{\frac{x+2-3(x-3)}{x-3}} = \frac{x+2+2x-6}{x+2-3x+9} = \frac{3x-4}{-2x+11}$

Domain $\{x|x \neq 3, \frac{11}{2}\}$ check both denominators

14) If $f(x) = 4x^3 - 4x^2 + 2x - 1$ and $g(x) = 3$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Find f°g(x)

$4(3)^3 - 4(3)^2 + 2(3) - 1 = 77$

Find g°f(x)

3 (has no x to substitute in for)

15) In a certain city, deforestation occurs at the rate of about 5.1% per year. If x is the total forested area of the city at the start of 2000, then $f(x) = 0.864x$ is the amount of forested land in the city at the start of 2001. Find and interpret $(f \circ f)(x)$. Find and interpret $(f \circ f \circ f)(x)$.

0.864(0.864x)

.....

$(f \circ f)(x) =$ .746x (Type an integer or decimal rounded to three decimal places as needed.)

The function $(f \circ f)(x)$ is the amount of forested land in the city at the start of 2002.

$(f \circ f \circ f)(x) =$ .645x (Type an integer or decimal rounded to three decimal places as needed.)

0.746(0.864x)

The function $(f \circ f \circ f)(x)$ is the amount of forested land in the city at the start of 2003.

16) The regular price of a computer is x dollars. Let $f(x) = x - 150$ and $g(x) = 0.8x$.

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**a.** Describe what the functions f and g model in terms of the price of the computer.

The function f gives the price of the computer after a $150 discount .

The function g gives the price of the computer after a 20% discount .

**b.** Find $(f \circ g)(x)$ and describe what this models in terms of the price of the computer.

$(f \circ g)(x) =$ .8x − 150

Plug g into f

The function $(f \circ g)(x)$ models the price of the computer after first a 20% discount and then a $150 discount .

g(x) 0.80 (20% off) then f(x) x-150

**c.** Find $(g \circ f)(x)$ and describe what this models in terms of the price of the computer.

$(g \circ f)(x) =$ .8x − 120

plug f into g 0.8(x-150)

The function $(g \circ f)(x)$ models the price of the computer after first a $150 discount and then a 20% discount .

f(x) x-150 then g(x) 0.80 (20% off)

**d.** Which composite function models the greater discount on the computer, $f \circ g$ or $g \circ f$? Why?

A. $f \circ g$ models the greater discount, since the 20% discount is taken on the regular price first.

17) A stone is thrown into a pond, creating a circular ripple that spreads over the pond in such a way that the radius is increasing at a rate of 2.6 ft/sec.

Complete parts **a** through **c**.

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**a)** Find a function for the radius in terms of t.

$r(t) =$ 2.6t

(Use integers or decimals for any numbers in the expression.)

**b)** Find a function A(r) for the area of the ripple in terms of the radius r.

$A(r) =$ $\pi r^2$ Area of a circle formula

(Type an exact answer, using $\pi$ as needed.)

**c)** Find $(A \circ r)(t)$. Plug 2.6 in for r $\Pi(2.6t)^2$

$(A \circ r)(t) =$ $6.76\pi t^2$

(Simplify your answer. Type an exact answer in terms of $\pi$. Use integers or decimals for any numbers in the expression.)

Explain the meaning of the previous function.

Choose the correct answer below.

A. The function gives the area of the ripple in terms of t.

18) A manufacturer of tools, selling rechargeable drills to a chain of home improvement stores, charges $4 more per drill than its manufacturing cost, m. The chain then sells each drill for 170% of the price that it paid the manufacturer. Find a function P(m) for the price at the home improvement stores. 1.7(x+4)

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The function for the price at the home improvement stores is given by

$P(m) =$ 1.7m + 6.8 .

(Simplify your answer. Use integers or decimals for any numbers in the expression. Do not include the $ symbol in your answer.)